

# Quadratic Equations

ALGEBRA



# Introduction

Quadratic equations are a fundamental concept in algebra that frequently appears on the GMAT. Understanding how to solve and manipulate quadratic equations is crucial for success in the quantitative section of the exam.

This guide will provide a thorough overview of quadratic equations, covering their definition, common forms, solving methods, and applications in GMAT-style problems.

## Definitions

- **Quadratic Equation** - An equation that can be written in the form  $ax^2 + bx + c = 0$ .

For example,  $2x^2 + 3x + 2 = 0$  is a quadratic equation while  $3x + 2$  is not a quadratic equation.

- **Factoring** - The process of breaking apart of an equation into factors (or separate terms) such that when the separate terms are multiplied together, they produce the original equation.

For example,  $x^2 - x - 2 = (x+1)(x-2)$ . In this case, the equation  $x^2 - x - 2 = 0$  can be broken apart into two factors [i.e.,  $(x+1)(x-2) = 0$ ] such that when these two separate terms (i.e., factors) are multiplied together, the result is the original equation.

- **Root** - A solution to an equation such as a quadratic equation such that  $f(x) = 0$ .

For example, for the quadratic equation  $x^2 + x - 12 = 0$ ,  $x = 3$  and  $x = -4$  are both roots since  $f(3) = 3^2 + 3 - 12 = 0$  and  $f(-4) = (-4)^2 - 4 - 12 = 0$

$a^2 + b^2 = c^2$

- **Multiplication and Division (from left to right):** Multiplication and division are performed from left to right, with simultaneous operations performed in the order they appear.

Example:  $4 \times 2 \div 5 = 8 \div 5 = 1.6$

- **Addition and Subtraction (from left to right):** Addition and subtraction are performed from left to right, with simultaneous operations performed in the order they appear.

Example:  $3 + 2 - 1 = 5 - 1 = 4$

## Examples of Simple Quadratic Equations

Although all quadratic equations by definition fit the form  $ax^2 + bx + c = 0$ , the most common simple format for a quadratic equation is as follows:

$$x^2 + 6x + 9 = 0$$

$$(a = 1, b = 6, c = 9)$$

$$x^2 - 4x + 4 = 0$$

$$(a = 1, b = -4, c = 4)$$

$$x^2 + 2x - 35 = 0$$

$$(a = 1, b = 2, c = -35)$$

## Examples of Complex or Untraditional Quadratic Equations

While most quadratic equations do appear in the format that the above examples appeared in, this is by no means the only format for a quadratic equation. The following are also quadratic equations:

$$2x^2 - 6x + 10 = 0$$

$$(a = 2, b = -6, c = 10)$$

$$5x^2 - 20 = 0$$

$$(a = 5, b = 0, c = -20)$$

$$9x^2 = 81$$

$$(a = 9, b = 0, c = -81 \text{ since we could subtract 81 from both sides})$$

## Solving Quadratic Equations

There are two main methods for solving quadratic equations: factoring and the quadratic formula.

- **Factoring**

Factoring involves breaking down the quadratic equation into its constituent linear factors. This method is effective when the quadratic expression can be easily decomposed into two binomials.

**Example:**

Solve the quadratic equation:  $2x^2 + 3x + 2 = 0$

**Solution:**

Factoring the quadratic equation, we get:

$$(x + 2)(x + 1) = 0$$

**Therefore, the solutions are  $x = -2$  and  $x = -1$ .**

$$a^2 + b^2 = c^2$$

- **Quadratic Formula**

The quadratic formula is a universal equation that can be used to solve any quadratic equation, regardless of its form. The formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, and c are the coefficients from the standard form of the quadratic equation.

**Example:**

Solve the quadratic equation:  $x^2 + 6x + 8 = 0$

**Solution:**

Using the quadratic formula, we find the solutions:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2(1)}$$

**$x = -4$  or  $x = -2$**

$$a^2 + b^2 = c^2$$

# COMMON GMAT PROBLEMS

## Problem 1

A rectangular garden has a length of  $x$  meters and a width of  $(x - 10)$  meters. If the area of the garden is 160 square meters, find the value of  $x$ .

### Solution:

**Step 1:** Set up the equation for the area of the garden.

The area of a rectangle is given by the formula length  $\times$  width. In this case, the length of the garden is  $x$  meters and the width is  $(x - 10)$  meters. Therefore, the area of the garden can be expressed as  $x(x - 10)$ .

**Step 2:** Set the area of the garden equal to 160 square meters and solve for  $x$ .

We are given that the area of the garden is 160 square meters. Therefore, we can set up the equation:

$$x(x - 10) = 160$$

Expanding the left side of the equation, we get:

$$x^2 - 10x = 160$$

Rearranging the equation, we get:

$$x^2 - 10x - 160 = 0$$

**Step 3:** Solve the quadratic equation.

We can solve the quadratic equation using factoring or the quadratic formula. In this case, we will use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a^2 + b^2 = c^2$$

where a, b, and c are the coefficients of the quadratic equation. Substituting the values of a, b, and c from the quadratic equation, we get:

$$x = (10 \pm \sqrt{(10^2 - 4(1)(-160))}) / (2(1))$$

$$x = (10 \pm \sqrt{740}) / 2$$

$$x = (10 \pm \sqrt{4 \cdot 185}) / 2$$

$$x = (10 \pm 2\sqrt{185}) / 2$$

$$\mathbf{x = 5 \pm \sqrt{185}}$$

## **Problem 2**

The zeros of the quadratic equation  $x^2 - 5x + 6 = 0$  are 2 and 3. Find the equation of the quadratic function in factored form.

### **Solution:**

Factor the quadratic equation

The zeros of a quadratic equation are the values of x that make the equation equal to zero. In this case, the zeros of the equation  $x^2 - 5x + 6 = 0$  are 2 and 3. This means that the equation can be factored as  $(x - 2)(x - 3) = 0$ .

**Therefore, the equation of the quadratic function in factored form is  $(x - 2)(x - 3) = 0$ .**

## TIPS TO EXCEL

- **Master factoring and the quadratic formula:** Practice using both methods so that you can choose the most efficient method for each problem.
- **Recognize common quadratic patterns :** Pay attention to the different forms in which quadratic equations can be written, and be able to identify these patterns quickly.
- **Review the concept of inequalities with quadratic expressions :** This is a common topic on the GMAT, and understanding how to solve inequalities with quadratic expressions will help you tackle these problems effectively.
- **Develop a strong understanding of algebra basics.** This will provide you with a solid foundation for solving quadratic equations.
- **Use process of elimination to narrow down your answer choices:** This can be a helpful strategy for multiple-choice questions.

$$a^2 + b^2 = c^2$$